Learning and Stability
Learning and Memory

Ramón y Cajal, 19th century
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Neuron doctrine
von Waldeyer-Hartz (1891)

Ramón y Cajal, 19th century
James, 1890; Hebb, 1949; Martin et al., 2000; Martin and Morris, 2002; …
Learning and Memory

Synaptic Plasticity:
If the neurons $i$ and $j$ are active together the synaptic efficiency $w_{ij}$ increases.

Thus, the influence of neuron $j$ on the firing of neuron $i$ is enhanced.

James, 1890; Hebb, 1949; Martin et al., 2000; Martin and Morris, 2002; ...
Learning and Memory

Synaptic Plasticity:
If the neurons $i$ and $j$ are active together the synaptic efficiency $w_{ij}$ increases.

Cell Assembly:
A group of strongly connected neurons which tend to fire together can represent a memory item.

Recall

A later presented (partial) recall signal of the original learned input has to activate the same group of neurons.

James, 1890; Hebb, 1949; Martin et al., 2000; Martin and Morris, 2002; ...
Learning and Memory

Synaptic Plasticity: If the neurons $i$ and $j$ are active together the synaptic efficiency $w_{ij}$ increases.

Cell Assembly: A group of strongly connected neurons which tend to fire together can represent a memory item.

Synaptic Plasticity and Memory Hypothesis

Synaptic Plasticity $\approx$ Learning

James, 1890; Hebb, 1949; Martin et al., 2000; Martin and Morris, 2002; ...
Synaptic Plasticity: Hebbian

Classical Hebbian plasticity: \[ \dot{\omega} = \mu u v \]

Long-Term Potentiation (LTP)

What are the long-term dynamics of Hebbian Plasticity?
Learning and Stability
Synaptic Plasticity: Hebbian

linear neuron model:
\[ v = u \omega \]

Classical Hebbian plasticity:
\[ \dot{\omega} = \mu u v \]

stability analysis

The weights follow **divergent** dynamics

Long-Term Potentiation (LTP)

\[ u: \text{input} \]
\[ v: \text{output} \]
\[ \omega: \text{weight} \]
\[ \mu: \text{learning rate} \]
Mechanisms of stabilization

Sliding threshold: 
\[ \frac{dw}{dt} = \mu \, v \, u \, (v - \Theta) \quad \mu << 1 \]
\[ \frac{d\Theta}{dt} = v \, (v^2 - \Theta) \quad v < 1 \]

Subtractive normalization: 
\[ \frac{1}{\mu} \frac{dw}{dt} = v \, u - v(n \cdot u) \frac{n}{N} \]

Multiplicative normalization: 
\[ \frac{dw}{dt} = \mu \left( v \, u - \alpha \, v^2 \, w \right), \quad \alpha > 0 \]
Sliding Threshold

BCM- Rule

\[ \frac{dw}{dt} = \mu \, vu \, (v - \Theta) \quad \mu << 1 \]

As such this rule is again unstable, but BCM introduces a sliding threshold

\[ \frac{d\Theta}{dt} = v \, (v^2 - \Theta) \quad v < 1 \]

Note the rate of threshold change \( v \) should be faster than then weight changes (\( \mu \)), but slower than the presentation of the individual input patterns. This way the weight growth will be over-dampened relative to the (weight – induced) activity increase.
less input leads to shift of threshold to enable more LTP

Time scales between learning rule and experiment are different

open: control condition
filled: light-deprived

Kirkwood et al., 1996
Mechanisms of stabilization

Sliding threshold:
\[
\frac{dw}{dt} = \mu \, v \, u \, (v - \Theta) \quad \mu \ll 1
\]
\[
\frac{d\Theta}{dt} = v \, (v^2 - \Theta) \quad v < 1
\]

Subtractive normalization:
\[
\frac{1}{\mu} \frac{dw}{dt} = vu - \frac{v(n \cdot u)n}{N}
\]

Multiplicative normalization:
\[
\frac{dw}{dt} = \mu \left( vu - \alpha \, v^2 \, w \right), \ \alpha > 0
\]
Subtractive normalization

Subtractive:

\[
\frac{1}{\mu} \frac{d\mathbf{w}}{dt} = \nu \mathbf{u} - \frac{\nu (\mathbf{n} \cdot \mathbf{u}) \mathbf{n}}{N}_{\nu <u>}
\]

With \( N \), number of inputs and \( \mathbf{n} \) a unit vector (all “1”). This yields that \( \mathbf{n} \cdot \mathbf{u} \) is just the sum over all inputs.

This normalization is *rigidly* apply at each learning step. It requires global information (info about ALL inputs), which is *biologically unrealistic*. 
Mechanisms of stabilization

Sliding threshold:
\[
\frac{dw}{dt} = \mu \, v \, u \, (v - \Theta) \quad \mu \ll 1
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\frac{d\Theta}{dt} = v \, (v^2 - \Theta) \quad v < 1
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Subtractive normalization:
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\frac{1}{\mu} \frac{dw}{dt} = v \, u - \frac{v (n \cdot u) n}{N}
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Multiplicative normalization:
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\frac{dw}{dt} = \mu \, (v \, u - \alpha \, v^2 \, w), \quad \alpha > 0
\]
Multiplicative normalization:

\[
\frac{dw}{dt} = \mu \left( vu - \alpha v^2 w \right), \quad \alpha > 0
\]

(Oja’s rule, 1982)

This normalization leads to an asymptotic convergence of \(|w|^2\) to \(1/\alpha\).

It requires only local information (pre-, post-syn. activity and the local synaptic weight).
Mechanisms of stabilization

Sliding threshold:
\[
\frac{dw}{dt} = \mu \, v \, u \, (v - \Theta) \quad \mu \ll 1
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\frac{d\Theta}{dt} = v \, (v^2 - \Theta) \quad v < 1
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Subtractive normalization:
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\frac{1}{\mu} \frac{dw}{dt} = v \, u - \frac{v(n \cdot u) \, n}{N}
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Multiplicative normalization:
\[
\frac{dw}{dt} = \mu \left( vu - \alpha \, v^2 w \right), \quad \alpha > 0
\]

Synaptic Scaling (Turrigiano et al., 1998) could be a candidate to stabilize plasticity
Synaptic Scaling

Activity changes compared to control

Number of AMPA receptors change

Synaptic Scaling

EPSPs

Control

TTX

Bicuculline

10 pA
1 s

10 pA
10 ms

Amplitude (pA)

Control amplitude (pA)

TTX

BIC

Synaptic Scaling


Diagram showing the relationship between control amplitude and firing rate, with a green line labeled TTX and an orange line labeled BIC.
Synaptic Scaling guarantees stable weight dynamics

\[ \dot{\omega} = \gamma [v_T - v] \]

*\( v \): output
*\( \omega \): weight
*\( v_T \): target firing rate
*\( \gamma \): learning rate
Hebbian Plasticity and Scaling

- **Hebbian Plasticity**
  \[ \dot{\omega} = \mu u v \]

- **Hebbian Plasticity + Synaptic scaling**
  \[ \dot{\omega} = \mu u v + \gamma [v_T - v] \omega^2 \]
Synaptic Plasticity: STDP

Makram et al., 1997
Bi and Poo, 2001
Synaptic Plasticity: Diversity

Kampa et al., 2007
Synaptic plasticity together with synaptic scaling is globally stable regardless of learning rule and neuron model.

Synaptic scaling is an appropriate candidate to guarantee stability.
Another learning mechanism

Structural Plasticity
Structural plasticity of axon terminals

In-vivo 2 Photon-Laser Imaging from the cortex of living mice reveals a permanent axonal remodelling even in the adult brain leading to synaptic rewiring.

Top: Axonal outgrowth/retraction
Red arrow: Axonal outgrowth
Yellow: Remains
Blue: Retracts

Bottom: Rewiring
Blue: Retracts and looses synapse
Red: Grows and creates new synapse.
Structural plasticity of **dendritic** spines

Dendritic spines on an appical dendrite on a LIV-neuron in V1 of the rat


Spine growth precedes synapse formation

**Spine formation** via filopodia-shaped spines (see arrow, top figure) precedes **synapse formation**. Spines in synapses are rather mushroom-shaped and carry receptor plates (active zones, red, top figure). Spines contact axonal terminals or axonal varicosities in reach and form synapses (left).

Knott et al., 2006
Stable and transient spines

Spines are highly flexible structures that are responsible (together with axonal varicosities) for synaptic rewiring. Only one third of all spines are stable for more than a month. Another third is semistable, meaning that it is present for a couple of days. Transient spines appear and disappear within a day.

In-vivo imaging of dendritic trees within the barrel cortex of living rats

Trachtenberg et al. (2002) Nature
Structural Plasticity: More abstract

Structural plasticity creates and deletes synapses
Structural Plasticity and Learning

Xu et al., 2009; Yang et al., 2009; Ziv and Ahissar 2009

Poor mice before training show structural changes after training.
Structural Plasticity and Learning

new spine formation

old spine elimination

Xu et al., 2009; Yang et al., 2009; Ziv and Ahissar 2009
Structural plasticity

Dendrite of an adult mouse in the visual cortex

Hofer et al., 2009

Kater et al., 1989; Mattson and Kater, 1989

monocular deprivation

Dendritic Outgrowth

Dendritic Regression

low activity

high activity

Kater et al., 1989; Mattson and Kater, 1989
Neuronal activity changes the intracellular calcium. Via changes in intra-cellular calcium, neurons change their morphology with respect to their axonal and dendritic shape. This leads to changes in neuronal connectivity which, in turn, adapts neuronal activity. The goal is that by these changes neurons achieve a homeostatic equilibrium of their activity.
**The model**

Circular axonal and dendritic probability spaces

Overlap determines the synaptic connectivity.
Individual shrinkage and growth following a homeostatic principle.

\[
\frac{dd_i}{dt} = -\gamma_{den} (c_i - c_{target})
\]

\[
\frac{da_i}{dt} = \gamma_{axo} (c_i - c_{target})
\]

Calcium dependent dendritic and axonal growth or shrinkage for synapse generation or deletion
The model (without the equations)

Homeostatic Process

- Normal Input
- Less Input
- Less Synapses at THIS neuron

- High Calcium
- Axonal Growth & Dendr. Shrinkage

Graph:
- Firing rate vs. Synaptic drive
- Target firing rate

Inputs:
- \( \text{input} \)
- \( \text{syn.den. } s_{ij} \)
- \( \text{axon j - dendrite i} \)

Outputs:
- \( \text{a}_i \)
- \( \text{d}_i \)
Memory

- Input
- Plasticity + Scaling
- Structural Plasticity

Strong, interconnected cluster representing memory
Working memory

Short-term memory

Activity

Hebbian plasticity

Synaptic Scaling

Long-term memory

Structural plasticity

Relations between biological processes and learning/memory